CALCULATION OF THE NONSTATIONARY TEMPERATURE FIELD IN A TWO-LAYER PLATE WHEN HEATED BY A MOVING SOURCE IN THE PRESENCE OF INHOMOGENEITIES ON THE CONTACT SURFACE OF THE LAYERS

A. A. Yanshin, Yu. A. Popov, and S. D. Tseitlin

UDC 536.21

The nonstationary two-dimensional problem of the theory of heat conduction in a two-layer plate with a separated section is solved in the case of heating one of the surfaces by a normally distributed source.

To investigate parameters which ensure the most effective determination of the quality of components by the active thermal method, we consider the nonstationary two-dimensional problem of heat conduction for a two-layer plate with a defect in the form of a separated portion on the boundary of the layers when one surface of the plate is being heated by a moving normally distributed source.

Statement of the Problem

An idealized model, with a good accuracy, describing the basic physical processes in the case of a thermal check, represents a two-layer plate of length L (Fig. 1) and thickness $H_1 + H_2$ formed of two layers of materials with densities ρ_1 and ρ_2, specific heats c_1 and c_2, and thermal conductivities λ_1 and λ_2. The homogeneous layers have everywhere an ideal thermal contact (in the $y = H_1$ plane) which is disturbed at a certain distance from the edge (from $x = x_1$ to $x = x_1 + D$), where an infinitely thin isolating layer of dimension D is located. On the upper surface of the plate there moves, with velocity v, a heat source which ensures a passage through this surface of a heat flux with a certain distribution

$$q = q_0 \exp[-k(x_0 + x - vt)^2]. \tag{1}$$

The choice of such a distribution of the heat flux from the source into the body corresponds to the actual conditions of a thermal check of two-layer components in the mode of scanning their surface. In this case the

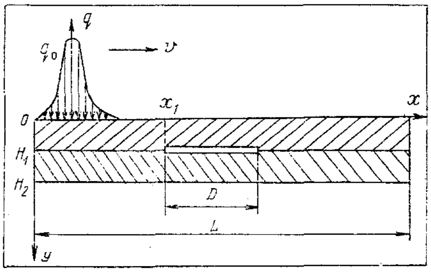

Fig. 1. The model of a two-layer plate with a separation region being considered.

Moscow Institute of Electronic Machine Construction. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 31, No. 4, pp. 721-726, October, 1976. Original article submitted September 30, 1975.

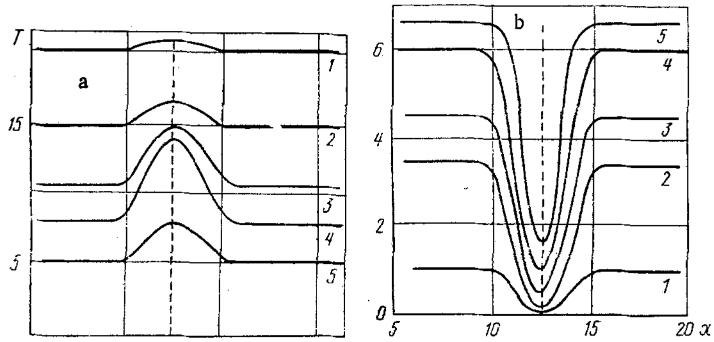

Fig. 2. Temperature profiles on the surface of a plate, obtained in the moving coordinate system for different values of the delay time τ_3 (T, °C; x, cm); a) the case of the surface being heated: 1) τ_3 = 20; 2) 40; 3) 80; 4) 160; 5) 380 sec; b) the case of the opposite surface: 1) τ_3 = 40; 2) 80; 3) 120; 4) 170; 5) 200 sec.

source and the radiometer, rigidly connected with one another, move with a constant velocity along the surface of the component. In view of a number of special features of thermal check, in a majority of cases the heating is carried out by an extended strip source; at the same time, the distribution of heat flux, for sources used in reality, into the body in the direction of scanning is described by a normal law [1]. Moving after the source, the radiometer registers the temperature profile of the loaded or the opposite surface of the component. The defects existing in the component are revealed from the anomalies of the surface temperature distribution being registered.

The basic equation, describing the distribution in such a body, in the Cartesian (x, y, t) coordinate system has the form

$$c(x, y)\rho(x, y)\frac{\partial T}{\partial t} = \lambda(x, y)\frac{\partial^2 T}{\partial x^2} + \frac{\partial}{\partial y}\left[\lambda(x, y)\frac{\partial T}{\partial y}\right] \tag{2}$$

for x, y belonging to the region $G = (0 \leq x \leq L,\ 0 \leq y \leq H_1 + H_2)$.

The boundary conditions have the form

$$\frac{\partial T}{\partial p} = f(x, y, t) \tag{3}$$

for x, y belonging to the boundary points $(\bar{G}/G)$.

On the boundary separating two materials (with the exception of the defect region) the condition

$$\lambda_1\left(\frac{\partial T}{\partial p}\right)_1 = \lambda_2\left(\frac{\partial T}{\partial p}\right)_2 \quad \text{for } T_1 = T_2 \tag{4}$$

must be fulfilled. On the defect portion we assume the condition of ideal isolation

$$\left(\frac{\partial T}{\partial p}\right)_1 = \left(\frac{\partial T}{\partial p}\right)_2 = 0. \tag{5}$$

We represent the function f(x, y, t) in the form

$$f(x, y, t) = \begin{cases} 0 & \text{everywhere on the boundary except } 0 \leqslant x \leqslant L,\ y = 0; \\ \dfrac{q_0}{\lambda_1}\exp[-k(x_0 + x - vt)^2] & \text{for } 0 \leqslant x \leqslant L,\ y = 0. \end{cases} \tag{6}$$

Such boundary conditions reflect the heating process of the component being checked, by a source moving along the surface with velocity v, with a normal distribution of flux into the body; this corresponds to the actual process of thermal check of a wide class of two-layer components. When solving the problem we neglect heat exchange between the body and the surrounding medium over the remaining portions.

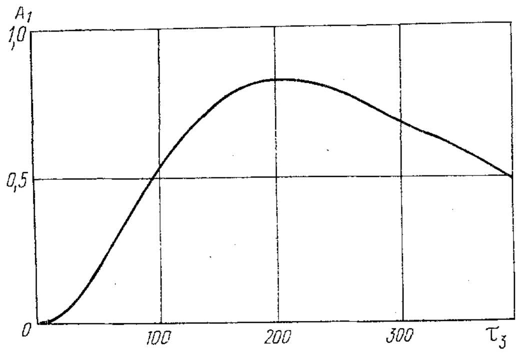

Fig. 3

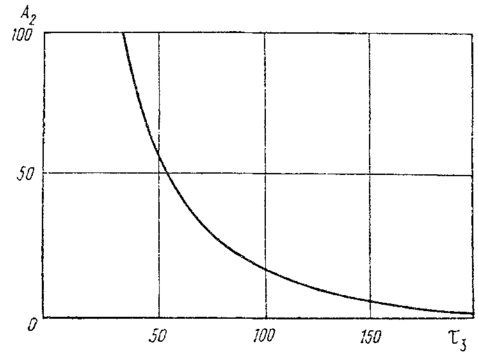

Fig. 4

Fig. 3. Dependence of the discovery criterion $A_1 = \Delta T/T$ for the surface being heated on the delay time $\tau_3 = L/v$ (τ_3, sec).

Fig. 4. Dependence of discovery criterion A_2 for the surface opposite to that being heated on the delay time τ_3 (τ_3, sec). $A_2 = |\Delta T|/T - |\Delta T|$.

In the role of the initial condition we choose the following:

$$T(x,\ y,\ t=0) = T^0 = \text{const}.$$

In our case it was assumed that $T^0 = 0$.

Numerical Algorithm for Solving the Problem

The problem under consideration was solved numerically. In the role of a numerical algorithm for realizing the solution of the problem on a digital computer, we applied the method of fractional steps, with a locally one-dimensional scheme [2, 3]. Here the space (x, y, t) was replaced by a grid space $\omega_h(ih_1, jh_2, \tau n)$, while the sought continuous function T(x, y, t) was replaced by the grid functions T_{ij}^n. Since the coefficient of thermal conductivity, density, and specific heat in our case are represented by piecewise-continuous functions of the coordinates $\{[\lambda(x, y), \rho(x, y), c(x, y)] \in Q^0\}$, for the numerical solution it was necessary to choose a divergent scheme, i.e., a scheme which would not violate, because of a break in the functions $\lambda(x, y)$, $\rho(x, y)$, $c(x, y)$, the conservation laws of the phenomenon under consideration.

After making Eq. (2) dimensionless, breaking it up according to the computation scheme chosen, and replacing it by the difference analog [3], the problem reduced to the solution of the algebraic system

$$\frac{T_{ij}^{n+1/2} - T_{ij}^n}{\tau} = \varkappa_{ij}\frac{T_{i-1,j}^{n+1/2} - 2T_{ij}^{n+1/2} + T_{i+1,j}^{n+1/2}}{h_1^2},$$
$$\frac{T_{ij}^{n+1} - T_{ij}^{n+1/2}}{\tau} = \frac{1}{h_2}\left(a_{i,j+1}\frac{T_{i,j+1}^{n+1} - T_{ij}^{n+1}}{h_2} - a_{ij}\frac{T_{ij}^{n+1} - T_{i,j-1}^{n+1}}{h_2}\right), \tag{7}$$

where

$$a_{ij} = \left[\frac{1}{h_2}\int_{i,j-1}^{i,j}\frac{dy}{\varkappa(x,\ y)}\right]^{-1},$$

$\varkappa(x, y)$ is the coefficient of thermal diffusivity, and $T_{ij}^{n+1/2}$ is the value of the temperature at the grid point (ih_1, jh_2) at the time instant $t_n + \tau/2$.

Grouping the terms of Eqs. (7), we can represent them in the form of the algebraic system of equations

$$A_{ij}T_{i-1,j}^{n+1/2} - B_{ij}T_{ij}^{n+1/2} + C_{ij}T_{i+1,j}^{n+1/2} = -D_{ij},$$
$$E_{ij}T_{i,j-1}^{n+1} - F_{ij}T_{ij}^{n+1} + G_{ij}T_{i,j+1}^{n+1} = -I_{ij}, \tag{8}$$

where

$$A_{ij} = \frac{\varkappa\tau}{h_1^2};\quad B_{ij} = \frac{2\varkappa\tau}{h_1^2};\quad C_{ij} = A_{ij};\quad D_{ij} = T_{ij}^n;$$

$$E_{ij}=\frac{a_{ij}\tau}{h_2^2};\ F_{ij}=\frac{\tau}{h_2^2}(a_{i,j+1}+a_{ij});\ G_{ij}=\frac{a_{i,j+1}\tau}{h_2^2};\ I_{ij}=T_{ij}^{n+\frac{1}{2}}.$$

The solution of the system (8) was carried out by the standard trial-run method, with the direction of trial-run being altered in succession.

The scheme chosen allows us to obtain the following accuracy of solving the numerical problem:

$$\varepsilon(x,\ y)\approx \|T(x,\ y,\ t)-T_{ij}^n\|_G=A(h_i^2+\tau),$$

where A is found, for example, by the method of subdividing the step h or by means of an a priori estimate [3].

The calculations were carried out for various values of L, H_1, H_2, D, λ_i, c_i, ρ_i, q_0, k, v, x_0. For example, in one of the cases we took L = 25 cm; $H_1 = H_2 = 0.5$ cm; D = 1.0 cm; $\lambda_1 = 0.41$ W/m·deg; $\rho_1 = 1.84\cdot10^3$ kg/m^3; $c_1 = 1.44\cdot10^3$ J/kg·deg; $\lambda_2 = 0.22$ W/m·deg; $\rho_2 = 1.12\cdot10^3$ kg/m^3; $c_2 = 1.43\cdot10^3$ J/kg·deg; $q_0 = 3.0\cdot10^4$ W/m^2; $v = 1.0\cdot10^{-2}$ m/sec; $k = 1.0\cdot10^3$ m^{-2}; $x_0 = -4\cdot10^{-2}$ m.

When considering the case of thermal check of two-layer components by means of an ID radiometer rigidly fixed to the moving source, which is one of the most effective methods of check [1], we have to analyze the temperature distribution in a moving coordinate system whose origin is located at the center of the heating spot. Such distributions on the surface being heated in the case described above are presented in Fig. 2a for points of the moving abscissa axis at various distances L' due to delay from the center of the heating spot. Eliminating the conditions of discovery of separated parts for the component, when the source and the radiometer are located on the same side, from the criterion $A_1 = \Delta T/T_0 = (T_1 - T_0)/T_0$ (T_1 is the temperature of the surface above the separated part; T_0 is the temperature of the surface above defect-free parts) (Fig. 3), we find that the most effective discovery of separation, for the parameters of thermal check considered, is possible when the delay time $\tau_3 = L'/v$ is about 200 sec.

Such temperature profiles, calculated for the case of two-sided check, i.e., when the radiometer is located on the other side from the source of heat energy, are presented in Fig. 2b for different τ_3. Estimating the conditions of discovery of defects in this case according to the criterion $A_2 = |\Delta T|/(T_0 - |\Delta T|)$ (Fig. 4), we find that the discovery of defects is improved under the condition $\tau_3 \to 0$.

The results of the analysis show that both for one-sided and for two-sided checks the discovery of defects is improved as the coefficient of concentration of the source increases (Figs. 3 and 4).

The results of the solution of the problem enable us to carry out the analysis for the components being checked with various thermophysical characteristics of the constituents, with variation of the thickness of each of the layers and dimensions of the defect. Carrying out the calculation under the conditions v = 0, $x_0 = -L/2$ and k = 0, we have a possibility of analyzing the possibility of check in the case where the component has a uniform distribution of heating. This is of interest when the temperature field is registered by a high-speed thermoscope and a "Termoprofil'," instrument.

NOTATION

x, y	are the rectangular coordinates;
t	is the time;
q	is the specific heat flux;
q_0	is the maximum specific heat flux;
k	is the flow concentration coefficient;
x_0	is the abscissa of the initial position of the source;
T	is the temperature;
p	is the normal to the surface.

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